Exact soliton-on-plane-wave solutions for two-component Bose-Einstein condensates

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By means of the Darboux transformation, we obtain analytical solutions for a soliton set on top of a plane-wave background in coupled Gross-Pitaevskii equations describing a binary Bose-Einstein condensate. We consider basic properties of the solutions with and without the cross interaction [cross phase modulation (XPM)] between the two components of the background. In the absence of the XPM, this solutions maintain properties of one-component condensates, such as the modulation instability (MI); in the presence of the cross interaction, the solutions exhibit different properties, such as restriction of the MI and soliton splitting.

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I. INTRODUCTION

Since the realization of a Bose-Einstein condensate (BEC) in dilute alkali-metal atomic vapors, rapid development took place in this field. In particular, the experimental observation of dark [1,2] and bright [3,4] solitons in a BEC has drawn attention to various aspects of dynamics of nonlinear matter waves, such as vortex dynamics [5], soliton propagation [6], interference patterns [7], and modulational instability (MI) [8,9]. A single-species BEC can be well described by the nonlinear Schrödinger (NLS) equation, alias the Gross-Pitaevskii (GP) equation, which has been analyzed from different points of view [10]. Various effects have been observed in multicomponent BECs, such as separation of components in a binary mixture [11], pattern formation [12], and interference between two condensates [13]. Multicomponent BECs are described by coupled NLS or GP equations. In one of the first theoretical works on this topic, Pu and Bigelow studied properties of the ground state and collective excitation in this model by numerical simulations [14]. Much more work has been done for theoretical analysis of binary BEC mixtures, but it was chiefly dealing with models of repulsive condensates; see, e.g., Refs. [15,16] and references therein. Recently, multicomponent models were extended to deal with soliton dynamics in spinor BECs; see Refs. [17–20].

In this paper, we investigate coupled GP equations for two-component self-attractive BECs (ones with negative scattering lengths) and present a soliton solution built on top of a plane-wave background. In the absence of the cross interaction between plane waves in the two components [cross phase modulation (XPM)], the solution effectively keeps properties of the single-component BEC, such as the MI; in the presence of XPM, the solution exhibits different properties, such as restriction of the MI and soliton splitting.

II. ANALYSIS AND RESULTS

The two-component BEC with attractive interactions between atoms is described by coupled GP equations. In a normalized form, they are

$$i\frac{\partial \phi_j}{\partial t} = -\frac{1}{2}\frac{\partial^2 \phi_j}{\partial x^2} - g\left(\frac{a_{jj}}{a_{12}}|\phi_j|^2 + |\phi_{3-j}|^2\right)\phi_j, \quad j=1,2, \quad (1)$$

where a_{jj} and a_{12} are, respectively, the negative scattering lengths of intra- and interspecies atomic collisions, x is measured in (characteristic) units of $x_0 \equiv 1\ \mu$m, t is in units of $mx_0^2/\hbar$, ϕ_j $(j=1,2)$ is in units of $\sqrt{n_0}$ (n_0 is the maximum density in the initial distribution of the condensate), and the interaction constant is defined as $g=4\pi n_0 x_0^2|a_{12}|$.

Generally, the coupled equations (1) are not integrable. However, in the case of $a_{11}=a_{22}=a_{12}$ Eqs. (1) amount to an integrable Manakov system [21,22]. In this case, by means of the Darboux transformation [23,24], we generate an exact vectorial (two-component) solution which accounts for a soliton interacting with a plane wave:

$$\phi_j = A_j e^{i\varphi}\left(1+\frac{4i\eta G_1}{F}\right) + \frac{2\eta \bar{C}_j}{\sqrt{g}}\frac{G_2}{F}\exp\left(\frac{i}{2}\varphi\right), \quad (2)$$

where $j=1,2$, and F, G_1, and G_2 are given by

$$F = De^{\theta_1} + A^2 De^{-\theta_1} - 2iA^2(L-\bar{L})\sin\varphi_1 + Ce^{-\theta_2}, \quad (3)$$

$$G_1 = Le^{\theta_1} - A^2\bar{L}e^{-\theta_1} - |L|^2 e^{-i\varphi_1} + A^2 e^{i\varphi_1}, \quad (4)$$

$$G_2 = Le^{-(i/2)(\varphi_1+\varphi_2)+(\theta_1-\theta_2)/2} + A^2 e^{(i/2)(\varphi_1-\varphi_2)-(\theta_1+\theta_2)/2}; \quad (5)$$

$$\theta_1 \equiv M_I x + [(\xi-k)M_I + \eta M_R]t/2 - \theta_{10}, \quad (6)$$

$$\theta_2 \equiv \eta x + \eta\xi t + \theta_{10}, \quad (7)$$

$$\varphi_1 \equiv M_R x + [(\xi-k)M_R - \eta M_I]t/2 + \varphi_{10},$$

$$\varphi_2 \equiv \xi x + (\xi^2-\eta^2)t/2 - \varphi_{10},$$

$$\varphi \equiv kx + [g(A_1^2+A_2^2) - k^2/2]t,$$

with $L \equiv M_R+\xi+k+i(M_I+\eta)$, $M_R+iM_I \equiv \sqrt{(k+\xi+i\eta)^2+A^2}$, and $D\equiv|L|^2+A^2$, $A\equiv\sqrt{4g(A_1^2+A_2^2)}$, $C\equiv|C_1|^2+|C_2|^2$. Here

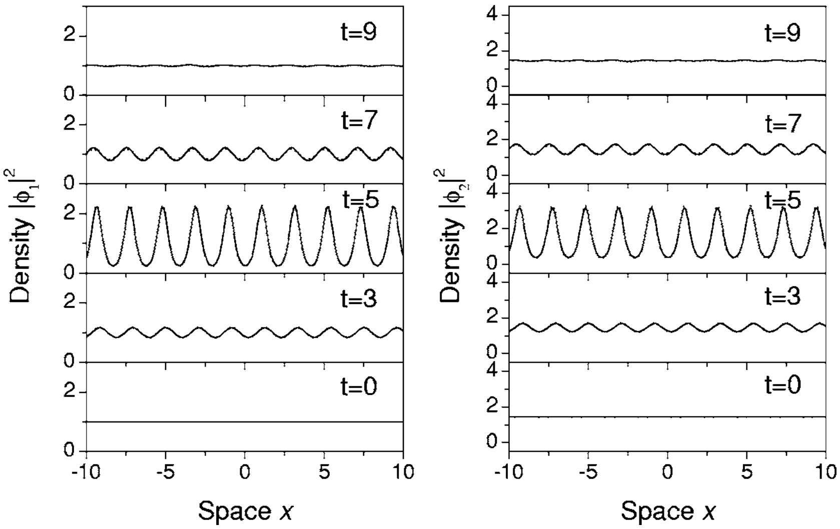

FIG. 1. A set of snapshots showing exact solution (8) by dotted curves, and numerical results, obtained under nonintegrable condition (15), by solid curves (in fact, the dotted and solid plots completely overlap). Parameters are $\eta=0.8$, $k=-0.1$, $g=1$, $A_1=1$, $A_2=1.2$, $\theta_{10}=6$ [the last corresponds to amplitude (13) of the initial perturbation triggering the onset of the modulational instability $\epsilon=2.4788\times10^{-3}$), and $\varphi_{10}=0$.

$k,\theta_{10},\varphi_{10}$ and A_1,A_2 are arbitrary real constants, and C_1 and C_2 are arbitrary complex constants subject to the constraint $A_1C_1+A_2C_2=0$. In the case of zero amplitude $A_1=A_2=0$, solution (2) amounts to $\phi_j=(\eta\epsilon_j/\sqrt{g})(\operatorname{sech}\theta)\exp(-i\varphi_2)$ where $\theta\equiv\eta(x+\xi)-\theta_0$, θ_0 is an arbitrary real constant, and ϵ_1 and ϵ_2 are arbitrary complex constants obeying the relation $|\epsilon_1|^2+|\epsilon_2|^2=1$. This result is consistent with that reported in Ref. [22], which was obtained by means of the inverse scattering transform. It corresponds to a stable vector soliton with velocity $V_{\mathrm{sol}}=-\xi$, width η^{-1}, and amplitudes $A_{s1}=|\eta\epsilon_1|/\sqrt{g}$, $A_{s2}=|\eta\epsilon_2|/\sqrt{g}$ that satisfy the relation $A_{s1}^2+A_{s2}^2=\eta^2/g$. The total norm of the vectorial soliton (proportional to the number of atoms in the binary condensate) is $Q\equiv Q_1+Q_2=\int_{-\infty}^{+\infty}(|\phi_1|^2+|\phi_2|^2)dx=2|\eta|/g$, with $Q_j=2|\eta||\epsilon_j|^2/g$. Further, its momentum and Hamiltonian are $M\equiv M_1+M_2=-(i/2)\int_{-\infty}^{+\infty}[(\bar{\phi}_1\phi_{1,x}-\phi_1\bar{\phi}_{1,x})+(\bar{\phi}_2\phi_{2,x}-\phi_1\bar{\phi}_{2,x})]dx=V_{\mathrm{sol}}Q$ and $H=(1/2)\int_{-\infty}^{+\infty}[(|\phi_{1,x}|^2+|\phi_{2,x}|^2)-g(|\phi_1|^2+|\phi_2|^2)^2]dx=M^2/(2Q)-(g^2/24)Q^3$.

On the other hand, when the soliton's amplitudes vanish, $A_{1s}=A_{2s}=0$ (e.g., $\eta=0$), solution (2) reduces to a plane wave of the form $\phi_1=A_1e^{i\varphi}$, $\phi_2=A_2e^{i\varphi}$ with amplitudes $A_{1,2}$, wave number k, and frequency $\Omega=g(A_1^2+A_2^2)-k^2/2$. Thus, in the general case the exact solution (2) represents a vectorial soliton embedded in a plane-wave background in two-component BECs. It should be noted that the condition $A_1C_1+A_2C_2=0$ determines the cross interaction between the two amplitudes of the plane-wave background. In fact, when $C_1=C_2=0$, the second term in expression (2) vanishes, then each soliton component is embedded in its own background, which realizes self-interaction in the two-component BEC; see a more detailed consideration of this special case below. If $C_1\neq0$ and $C_2\neq0$, the second term in solution (2) is different from zero, and then the above-mentioned condition $A_1C_1+A_2C_2=0$ implies that coefficients C_1 and C_2 depend on the plane-wave amplitudes A_2 and A_1, respectively, which is a manifestation of the cross interaction in the two-component solution.

To understand the behavior of solution (2), we first consider in more detail the above-mentioned case of $C_1=C_2=0$, when the solution (2) simplifies to

$$\phi_j=A_je^{i\varphi}(1-2\eta W), \tag{8}$$

$$W\equiv\frac{1}{A}\left(\frac{a\cosh\theta_1+\sin\varphi_1}{\cosh\theta_1+a\sin\varphi_1}-i\frac{b\sinh\theta_1+c\cos\varphi_1}{\cosh\theta_1+a\sin\varphi_1}\right), \tag{9}$$

with $a=-iA(L-\bar{L})/D$, $b=A(L+\bar{L})/D$, and $c=(A^2-|L|^2)/D$. Note that expressions (8) and (9) do not include θ_2 and φ_2,

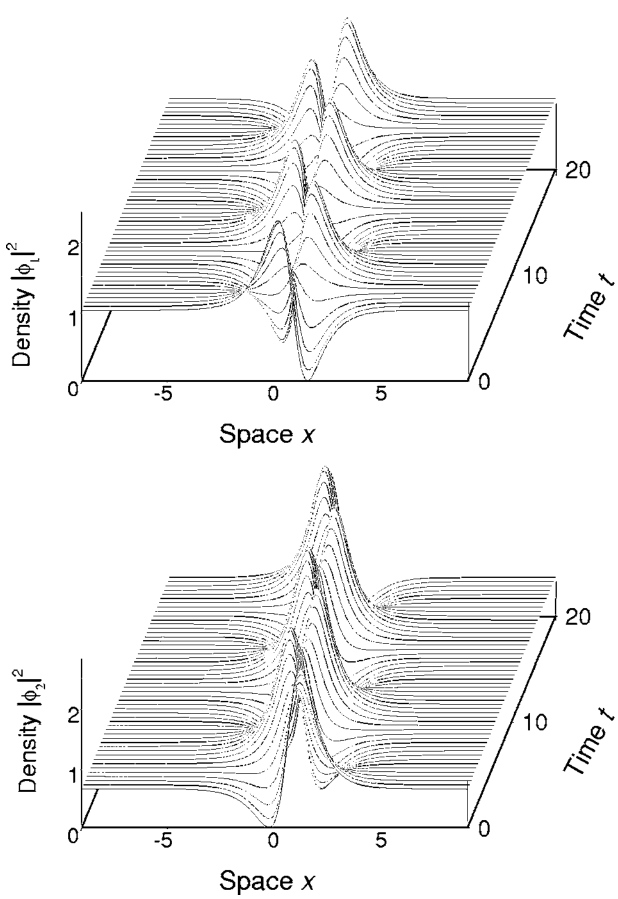

FIG. 2. Spatiotemporal distribution of densities of the two components in solution (16) (the phase difference between the components is π). Parameters are $k=-0.1$, $g=1$, $A_1=1$, $A_2=0.8$, $\theta_{10}=-2$, and $\varphi_{10}=0$.

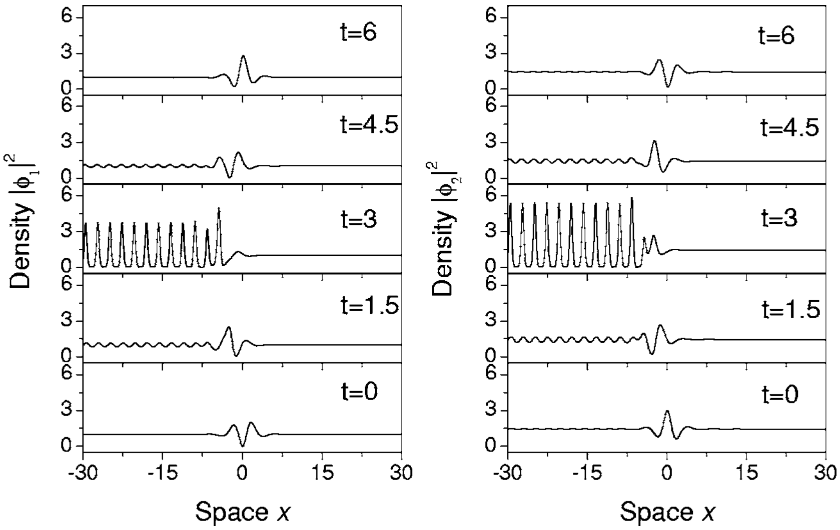

FIG. 3. A set of snapshots of exact solution (18) for $\eta=-1.5$, $k=-0.1$, $g=1$, $A_1=1$, $A_2=1.2$, $\theta_{10}=-6$, $\varphi_{10}=0$, and $\theta_{20}=-5$, $\varphi_{20}=0$.

and the two components share a common shape. This solution is similar to that for the single-component BEC. As $M_I\neq 0$, from Eq. (8) one can extract the total norm of the soliton component of the solution, which we define as

$$\int_{-\infty}^{+\infty}[(|\phi_1|^2-A_1^2)+(|\phi_2|^2-A_2^2)]dx=\frac{\eta(b^2+c^2)}{g|M_I|}I, \quad (10)$$

$$I\equiv\int_{-\infty}^{+\infty}\frac{\eta+A\cosh x\sin(Bx+\Delta)}{[\cosh x+a\sin(Bx+\Delta)]^2}dx, \quad (11)$$

with $\Delta=-\eta M_I(1+B^2)t/2+B\theta_{10}+\varphi_{10}$ and $B=M_R/M_I$. In deriving the above expression, use has been made of relations $|\phi_1(\pm\infty,t)|^2=A_1^2$ and $|\phi_2(\pm\infty,t)|^2=A_2^2$, which are background densities in the two-component condensate. It can be verified that integral (11) does not depend on Δ; hence the soliton's total norm is a conserved quantity.

When $M_I=0$, the soliton velocity vanishes, i.e., the plane wave completely traps the soliton. The creation of such a trapped soliton is actually a manifestation of the MI [25]. Setting, in particular, $\xi=-k$ and $A^2>\eta^2$ yields $M_I=0$, and W in solution (8) becomes

$$W=\frac{1}{A}\frac{\eta\cosh\theta_1+A\sin\varphi_1-iM_R\sinh\theta_1}{A\cosh\theta_1+\eta\sin\varphi_1}, \quad (12)$$

with $\theta_1\equiv\eta M_R t/2-\theta_{10}$ and $\varphi_1\equiv M_R(x-kt)+\varphi_{10}$, where $M_R=\sqrt{A^2-\eta^2}$, and θ_{10} and φ_{10} are arbitrary real constants. Expression (12) is periodic in x, with the period $\Lambda=2\pi/M_R$, and aperiodic in t. To better understand the MI development provided by this solution, we assume that

$$\epsilon\equiv\exp(-\theta_{10}) \quad (13)$$

is small (it then plays the role of a small amplitude of the initial perturbation that triggers the onset of the MI), and linearize the initial form of solution (8) (at $t=0$) in ϵ, taking Eq. (12) into account, which yields

$$\phi_j(x,0)\approx A_j e^{i\varphi}[\rho-\epsilon\chi\sin(M_R x+\varphi_{10})], \quad (14)$$

where $\rho\equiv(A^2-2\eta^2-i2\eta M_R)/A^2$ with $|\rho|^2=1$, $\chi\equiv 4\eta M_R(M_R-i\eta)/A^3$. Direct numerical simulations of the underlying equations (1) demonstrate that the evolution of

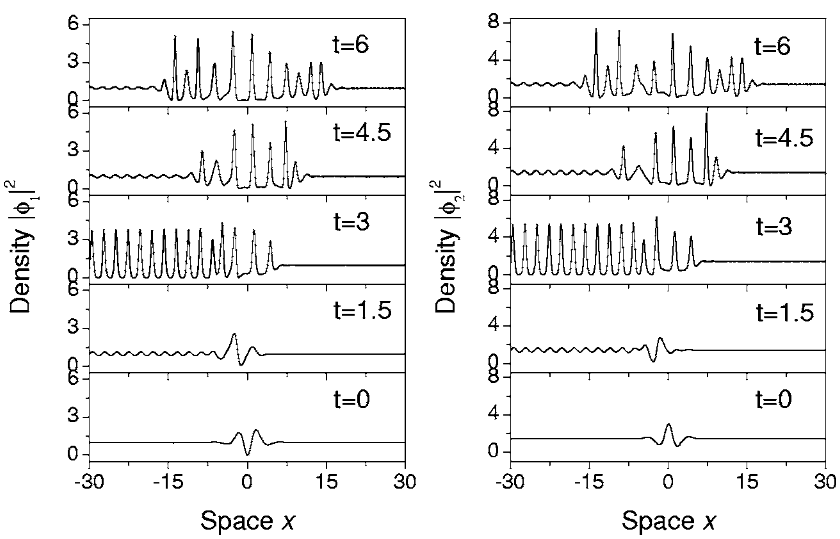

FIG. 4. The same as in Fig. 3, but obtained from a numerical solution of Eqs. (1) with the nonlinear constants taken as per Eq. (15). Other parameters are identical to those in Fig. 3.

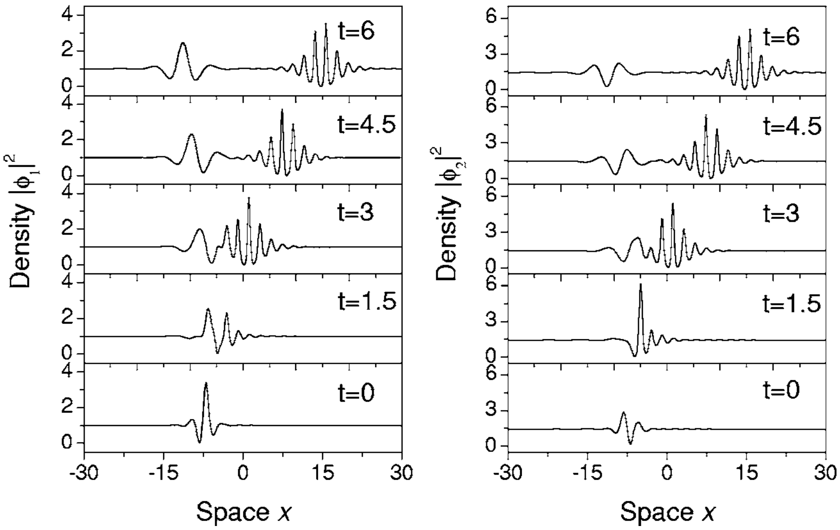

FIG. 5. A set of snapshots of the general-form analytical solution (2). Parameters are $\eta=1.5$, $\xi=-0.4$, $k=-0.5$, $g=1$, $A_1=1$, $A_2=1.2$, $\theta_{10}=5$, $\varphi_{10}=0$, $C_1=1$, and C_2 is determined by the constraint $A_1C_1+A_2C_2=0$.

initial configuration (14), which is a plane wave with a small modulational perturbation added to it, indeed closely follows the exact solution provided by Eqs. (8) and (12).

The above results were obtained under the Manakov integrability conditions $a_{11}=a_{22}=a_{12}$, which may not be exactly satisfied in reality. However, the actual difference of the scattering lengths in a BEC mixture of two different hyperfine states of the same atomic species is very small [26], and therefore the Manakov model may be used as a good approximation. For instance, taking

$$a_{11}=-1.03, \quad a_{12}=-1, \quad a_{22}=-0.97, \tag{15}$$

numerical solution of Eqs. (1) yields the picture of the MI development shown in Fig. 1. Comparing it with the exact solution obtained for $a_{11}=a_{22}=a_{12}$ demonstrates that the two solutions are virtually indistinguishable.

We now turn to solution (2) in a more general situation, with nonzero coefficients $C_{1,2}$ and $A_{1,2}$, subject to the requirement $A_1C_1+A_2C_2=0$ [recall that this condition is a part of the exact solution (2)], which will make it possible to explicitly consider cross-interaction (XPM) effects between two plane waves in the two-component BEC. In this case, we may set $C_1=4\sqrt{g}A_2C$ and $C_2=-4\sqrt{g}A_1C$, where C is the arbitrary complex constant. Further, we fix $\xi=-k$ in solution (2) and analyze two representative situations in detail.

(i) Setting $A^2=\eta^2$, i.e., $4(A_1^2+A_2^2)=A_{s1}^2+A_{s2}^2$, solution (2) can be written as

$$\phi_j=-e^{i\varphi}\left(A_j\tanh\frac{\theta_2}{2}+(-1)^j\sqrt{2}A_{3-j}\frac{\exp(i\varphi_3)}{\cosh(\theta_2/2)}\right), \tag{16}$$

where $\theta_2\equiv\eta(x-kt)+\theta_{10}$, $\varphi\equiv kx+(\eta^2/4-k^2/2)t$, $\varphi_3\equiv\eta^2t/8+\varphi_{10}$, and θ_{10} and φ_{10} are arbitrary real constants; the density distribution corresponding to this solution is $|\phi_1|^2+|\phi_2|^2=(A_2^2+A_1^2)[1+\mathrm{sech}^2(\theta_2/2)]$ (see Fig. 2). Solution (16) may be regarded as a superposition of bright and dark solitons, produced by the action of the XPM. In particular, the solution with $A_1=0$ or $A_2=0$ is a complex consisting of the bright and dark solitons in the first and second species, or vice versa. From the imposed condition $A^2=\eta^2$, it follows that $|\eta|=\sqrt{4g(A_1^2+A_2^2)}$, showing that the width of the soliton is controlled by the amplitude of the plane wave. In Fig. 2 one may observe a shift of the soliton's peak due to the action of the XPM.

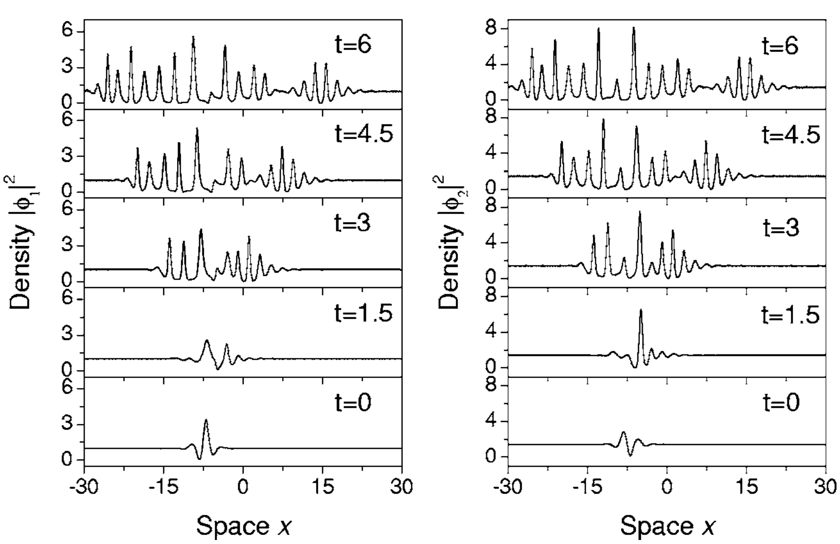

FIG. 6. The same as in Fig. 5, but for a numerically found solution of Eqs. (1) with $a_{11}=-1.03$, $a_{12}=-1$, $a_{22}=-0.97$. Parameters are identical to those in Fig. 5.

(ii) In the case when $A^2 > \eta^2$, i.e.,

$$4(A_1^2 + A_2^2) > A_{s1}^2 + A_{s2}^2, \tag{17}$$

solution (2) can be written as

$$\phi_j = A_j e^{i\varphi}(1 - 2\eta W_1) - (-1)^j 2\eta A_{3-j} W_2, \tag{18}$$

$$W_1 = \frac{1}{A}\frac{\eta \cosh \theta_1 + A \sin \varphi_1 - iM_R \sinh \theta_1}{A \cosh \theta_1 + \eta \sin \varphi_1 + Ae^{-\theta_2}}, \tag{19}$$

$$W_2 = \frac{1}{A}\frac{(M_R + i\eta)e^{(-i\varphi_1+\theta_1)/2} + Ae^{(i\varphi_1-\theta_1)/2}}{A \cosh \theta_1 + \eta \sin \varphi_1 + Ae^{-\theta_2}} e^{[-\theta_2+i(\varphi-\varphi_2)]/2}, \tag{20}$$

with $\theta_1 \equiv (\eta/2)M_R t - \theta_{10}$, $\theta_2 \equiv \eta(x-kt) + \theta_{20}$, $\varphi_1 \equiv M_R(x-kt) + \varphi_{10}$, $\varphi_2 \equiv -kx + \frac{1}{2}(k^2 - \eta^2)t - \varphi_{20}$, and $M_R = \sqrt{A^2 - \eta^2}$, while θ_{10}, θ_{20}, φ_{10}, and φ_{20} are arbitrary real constants. From expressions (19) and (20), one can see that $W_2 \to 0$ as $\theta_2 \to \pm\infty$, and $W_1 \to W$ as $\theta_2 \to +\infty$ [recall that W is given by expression (12)], and $W_1 \to 0$ as $\theta_2 \to -\infty$. Thus, taking into regard the form of solution (8) and expression (12), one may conclude that solution (18) describes *partial* MI, as the growth of the instability can be restrained in the limit of $\theta_2 \to -\infty$, as illustrated by Fig. 3. However, if the nonlinear constants a_{11}, a_{12}, and a_{22} slightly deviate from the integrable case $a_{11} = a_{12} = a_{22}$, a numerically found counterpart of solution (18) is conspicuously different from it (see Fig. 4).

Finally, we turn to solution (2) in the general case. From expressions (6) and (7), one can see that the solution contains terms with different velocities $V_1 = -\frac{1}{2}[(\xi - k) + \eta M_R / M_I]$ and $V_2 = -\xi$, respectively, which leads to splitting of the soliton part of the solution on top of the plane wave into two wave packets, high- and low-frequency ones, as shown in Fig. 5. Further, in Fig. 6, this exact analytical solution, obtained in the integrable case with $a_{11} = a_{12} = a_{22} = 1$, is compared with a numerically found solution for the generic nonintegrable version of Eqs. (1), with the normalized scattering lengths chosen as in Eq. (15). Figure 6 demonstrates that the soliton part of the numerically found solution, on top of the plane-wave background, also splits into two packets, a stable high-frequency and an unstable low-frequency component.

III. CONCLUSION

We have presented exact solutions for coupled Gross-Pitaevskii equations describing binary BECs, in the form of a soliton placed on top of a plane-wave background. It was shown that, when the intensity of the plane-wave background exceeds a quarter of the soliton's peak intensity, which is expressed by condition (17), the exact solution describes development of the modulational instability, which can be used in physical applications to generate a soliton train. Moreover, it was shown that this soliton train can also be generated even if parameters of the coupled equations do not exactly satisfy the corresponding integrability conditions. Also, we have discussed the cross interaction (XPM) of the plane-wave backgrounds in the two components of the system. It was shown that the XPM between two plane-wave amplitudes leads to a π phase difference between the components, and, in the most general case, it causes splitting of the soliton and formation of a complex pattern. In particular, under the same condition (17) as mentioned above, the XPM interaction between the two plane waves helps to effectively restrain the modulational instability.

It is relevant to mention that the coupled underlying equations (1) are also integrable in the case of $g < 0$ and $a_{11} = a_{22} = a_{12}$ (a mixture of self-repulsive condensates), and also, with either sign of g, for $a_{11} = a_{22} = -a_{12}$ [27], which corresponds to a mixture of two self-repulsive condensates that attract each other, or two self-attractive ones which repel each other. In the former case, when single-component bright solitons are impossible due to the intraspecies repulsion, so-called symbiotic solitons exists and are stable, being supported by the interspecies attraction [28]. However, direct attempts to construct bound states of plane waves and solitons by means of the Darboux transformation lead to singular solutions in these cases.

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